Detailed Proof of the Proposition 1

**Proposition 1**: A solution to model (3), where  , is suboptimal if any one of the following conditions hold: **1)**  and ; **2)**  and ; **3)** .

*Proof*: This proposition can be proven based on the reduction to absurdity method. First, we assume , . Let  be the Lagrange function of model (3) in the manuscript. Then, applying the Karush–Kuhn–Tucker (KKT) conditions for (3) yields the following.

 (4a)

 (4b)

 (4c)

The condition   ensures that  and . Introducing these conditions into (4a) and (4b), and merging (4a) and (4b) by eliminating the variable  yields the following.

 (4d)

Under **condition 1)**, we obtain  and , which ensures that  based on (4c). However, . Therefore, these conditions produce a contradiction in (4d) because the value of  is always positive.

Under **condition 2)**, we obtain  and , which ensures that . Again, these conditions produce a contradiction in (4d) because the value of is always positive.

Under **condition 3)**, we obtain the relationship  since ,  and , which is contrary to (4d).